

Generic Grading Instructions for Mathematical Exams with Batch Harmonization

These instructions define a structured grading protocol for mathematical exams using the official grading scheme and, where available, the official solution.

Each script must first be graded independently and completely. During this first pass, a consistency register should be maintained for recurring, borderline, or ambiguous cases. After the first pass, a batch-wide harmonization review may be performed. If a grading interpretation must be refined, the refined interpretation must be applied retroactively to all affected scripts. At all times, points may only be awarded for explicitly visible, mathematically correct, and form-compliant student work.

1. Execution mode.

All grading must follow a structured protocol. All relevant grading steps must be shown explicitly. Implicit reasoning and global judgments are not allowed.

2. Mandatory output structure.

For each task, exercise, or question, the response must contain:

- Step 1: Page Scan
- Step 2: Detection Map
- Step 3: Object Verification
- Step 4: Rubric-by-Rubric Grading
- Step 5: Rubric-Item Summary
- Step 6: Subtotal
- Step 7: Consistency Notes (if relevant)

3. Batch correction process.

Each script must be corrected with rigorous, transparent grading. Equivalent cases must be treated consistently across scripts. If a batch-wide consistency adjustment is made, it must be applied to all affected scripts.

4. Reference and precedence rule.

The official grading scheme is binding. Task-specific scoring instructions in the grading scheme override general grading rules. Where available, the official solution defines the intended mathematical targets, but it does not override explicit scoring instructions. If the grading scheme and the official solution conflict or remain ambiguous, the item must be flagged for manual review rather than resolved by inference.

5. No-inference and visible-object rule.

A rubric item may only be awarded if the required object:

- is explicitly visible,
- appears in the required form,
- is mathematically correct.

No points may be awarded for implied, reconstructed, guessed, or inferred objects.

6. Authorship rule.

Only objects attributable to the student are eligible for grading. Annotations, corrections, highlights, or additions made by the grader must be ignored and must not be treated as student

work. If a required component appears only in grader annotations and not in the student's original work, the object is incomplete and does not satisfy the visibility requirement. If authorship is unclear, this must be stated explicitly.

7. Embedded-object clarification rule.

A required mathematical object is considered visible and eligible for points if it appears explicitly and correctly anywhere in the student's work relevant to the same task, including as part of an intermediate step, computation, derivation, or transformation, provided that:

- its structure is clearly identifiable,
- all required components are explicitly written,
- no interpretation, reconstruction, or inference is necessary.

Objects that are only partial, implied, or reconstructed do not satisfy the visibility requirement.

8. Default-zero and justification rule.

Each rubric item starts at zero. Points require:

- an explicit object,
- a clear page and location reference,
- a short justification.

9. Independence rule.

Each rubric item is evaluated independently, except where the follow-through rule explicitly applies.

10. Form rule.

Points require the object to appear in the required form, unless the official grading scheme explicitly allows credit despite a form defect.

11. Alternative-solution rule.

A mathematically correct alternative solution must receive the corresponding points if the required object is explicitly visible, mathematically correct, and satisfies the rubric criteria, even if the method differs from the official solution.

12. Follow-through / Folgefehler rule.

When an earlier rubric item contains an error, later rubric items must still be evaluated relative to the student's explicitly written previous result, provided that the previous result is mathematically identifiable. Award later points when the subsequent reasoning, calculation, or conclusion is correct as a consequence of the student's own carried-forward result.

Do not award follow-through points for:

- missing objects,
- unreadable or unidentifiable work,
- guessed or unsupported final answers,
- rubric items whose purpose is specifically to test the original setup, definition, classification, sketch, or identification,
- cases where the later work introduces a new independent error.

Any such award must be marked explicitly as “Folgefehler” or “follow-through” in the grading report.

- 13. Follow-through object clarification** If a later rubric item evaluates a method applied to a previously obtained object, the later item must be graded relative to the student’s visible previous object, even if that object is not the official one, provided the object is mathematically identifiable and was produced by a reasonable approach.

Do not deny method or computation points solely because the previous object is in the wrong form, unless the later rubric item explicitly tests that required form.

For kernel, nullspace, image, transformation, diagonalization, parametrization, equation-solving, or similar tasks, a visible matrix, vector, equation, function, or expression may serve as a valid follow-through object if it is clearly derived from the student’s earlier work. Later points are awarded when the student correctly applies the intended method to that object.

Form-specific rubric items remain strict. A wrong-form object may receive zero for the form-specific item, but it may still be used as the basis for later follow-through items.

- 14. Crossed-out work and multiple attempts rule.**

Crossed-out work is ignored unless it is the only clearly attributable attempt and the student’s final intention remains unambiguous. If multiple attempts are present, grade the final clearly indicated attempt. If no final attempt is clearly indicated, state the ambiguity explicitly and flag the item for manual review if necessary.

- 15. Detection phase.**

For each task:

- (a) scan all pages,
- (b) identify relevant objects,
- (c) classify each object as **VISIBLE**, **PARTIAL**, or **NOT FOUND**.

- 16. Classification rule.**

Use the following labels:

- **VISIBLE**: explicitly present, identifiable, and verifiable,
- **PARTIAL**: present but incomplete, illegible, or not fully verifiable,
- **NOT FOUND**: absent or only inferable.

- 17. Page scan protocol.**

For each relevant page, record:

- the page number,
- the main mathematical objects relevant to the task.

- 18. Detection map.**

For each relevant object, record:

- page and approximate location,
- object type,

- classification.

Approximate locations should use compact labels such as “top-left”, “middle”, or “bottom-right”.

19. Mathematical verification.

Verify only the mathematically relevant parts needed for the rubric decision. In particular:

- check decisive definitions, substitutions, transformations, intermediate results, and final results explicitly as needed,
- avoid unnecessary expansion of trivial steps,
- do not use vague statements such as “looks correct”.

20. No substitution rule.

Never replace a student object by the model solution. Grade only the student’s own visible work.

21. Rubric grading rule.

For each rubric item, state:

- the object with page and location,
- visible: yes/no,
- correct: yes/no,
- form correct: yes/no,
- follow-through: yes/no, if applicable,
- points,
- a short justification.

22. Mandatory rubric-item summary.

After each task, output an explicit summary of all rubric item identifiers and awarded points. Every rubric item must be listed individually, including zero-point items.

Identifiers may use letters, numbers, primes, or combinations thereof, and must be reproduced exactly as in the grading scheme.

Required format examples:

- Task X: $A = 1$, $B = 0.5$, $C = 0$
- Task Y: $1a = 1$, $1b = 0$, $1c = 2$
- Task Z: $D' = 1$, $E' = 0.5$, $F' = 0$

No awarded or non-awarded rubric item may be omitted from the summary.

23. Mandatory zero-point and follow-through check Before assigning 0 points to any rubric item, explicitly check whether the missing or incorrect object is the object directly tested by this rubric item, or whether it is only an earlier object used as input for a later method.

If the rubric item directly tests a required object, form, definition, classification, setup, or identification, grade it strictly.

If the rubric item tests a later method, computation, transformation, conclusion, or continuation based on a previous student-produced object, grade it relative to the student’s visible

previous object, provided that object is mathematically identifiable, reasonably derived from the student's work, and not merely guessed.

A previous object may receive 0 points for being wrong or in the wrong form, but it may still serve as a valid follow-through object for later rubric items.

When awarding such points, mark them explicitly as follow-through.

24. Binary vs. partial rule.

By default, structured objects or holistic targets are graded binarily:

- fully correct $\Rightarrow$ full points,
- otherwise $\Rightarrow$ 0 points.

An explicit partial-scoring rule in the grading scheme overrides this default and must be applied exactly as written. Partial credit is never inferred.

25. Sketch and diagram rule.

A sketch or diagram is graded by qualitative mathematical correctness, not by artistic precision. It must match the relevant structure, orientation, labeling, and qualitative properties required by the rubric.

26. Unclear work rule.

If handwriting, notation, structure, or authorship is unclear, this must be stated explicitly. Unclear work does not satisfy a rubric item unless the required object remains explicitly identifiable and verifiable.

27. Consistency and review rule.

Apply equivalent decisions identically across scripts. After grading, perform a consistency review across the batch and harmonize recurring cases. Any revised interpretation must be applied retroactively to all affected scripts.

28. Subtotal rule.

State the subtotal after each task.

Summary principle.

Points are awarded if and only if the required object is:

1. visible,
2. mathematically correct,
3. in the required form,

unless an explicit task-specific scoring rule states otherwise.